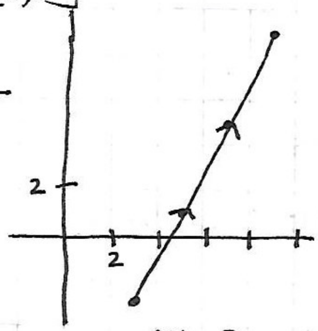
AP Calculus BC — Parametric Equation

1) a) $x = 2t+3 \quad y = 4t-3$

$t: [0,3]$

t	x	y
0	3	-3
1	5	1
2	7	5
3	9	9



$x-3 = 2t$

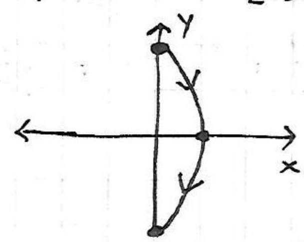
$t = \frac{x}{2} - \frac{3}{2}$

$y = 4\left(\frac{x}{2} - \frac{3}{2}\right) - 3$

$\boxed{y = 2x - 9;\ 3 \le x \le 9}$

b) $x = \sin t \quad y = 2\cos t \quad [0, \pi]$

t	x	y
0	0	2
$\pi/2$	1	0
π	0	-2



$\cos t = \frac{y}{2}$

$\sin^2 t + \cos^2 t = 1$

$x^2 + \left(\frac{y}{2}\right)^2 = 1$

$x^2 + \frac{y^2}{4} = 1$

2) a) $x = 4\sin t \quad y = 2\cos t$

$\frac{dx}{dt} = 4\cos t \quad \frac{dy}{dt} = -2\sin t$

$\frac{dy}{dx} = \frac{-2\sin t}{4\cos t} = -\frac{1}{2}\tan t$

$\frac{d^2y}{dx^2} = \frac{-\frac{1}{2}\sec^2 t}{4\cos t} = -\frac{1}{8}\sec^3 t$

b) $x = t^2 - 3t \quad y = t^3$

$\frac{dx}{dt} = 2t-3 \quad \frac{dy}{dt} = 3t^2$

$\frac{dy}{dx} = \frac{3t^2}{2t-3}$

$\frac{d^2y}{dx^2} = \dfrac{\dfrac{(2t-3)(6t) - 3t^2(2)}{(2t-3)}}{2t-3}$

$= \frac{6t(2t-3) - 6t^2}{(2t-3)^2} = \frac{6t^2 - 18t}{(2t-3)^2}$

c) $x = \ln(2t) \quad y = \ln(3t)^4$

$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = \frac{4(3t)^3 \cdot 3}{(3t)^4} = \frac{12}{3t} = \frac{4}{t}$

$\frac{dy}{dx} = \frac{4/t}{1/t} = 4$

$\frac{d^2y}{dx^2} = 0$

d) $x = \ln(5t) \quad y = e^{5t}$

$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 5e^{5t}$

$\frac{dy}{dx} = \frac{5e^{5t}}{1/t} = 5te^{5t}$

$\frac{d^2y}{dx^2} = \frac{5t \cdot (5e^{5t}) + 5e^{5t}}{1/t}$

$= \frac{25te^{5t} + 5e^{5t}}{1/t}$

$= 25t^2e^{5t} + 5te^{5t}$

$= 5te^{5t}(5t+1)$

3) $x = t^2 - 1 \quad y = e^{t^3}$

$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = 3t^2 e^{t^3}$

$\frac{dy}{dx} = \frac{3t^2 e^{t^3}}{2t} = \frac{3}{2} t e^{t^3}$

4) $x = t^3 \quad y = t^2 - 5t + 2$

$\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 2t - 5$

$\frac{dy}{dx} = \frac{2t-5}{3t^2}$

$\left.\frac{dy}{dx}\right|_{t=2} = -\frac{1}{12}$

$y + 4 = -\frac{1}{12}(x - 8)$

5) $x = 2 - 3\cos t \quad y = 3 + 2\sin t$

a) $\frac{dx}{dt} = 3\sin t \quad \frac{dy}{dt} = 2\cos t$

$\frac{dy}{dx} = \frac{2\cos t}{3\sin t}$

b) $x(\frac{\pi}{4}) = 2 - \frac{3\sqrt{2}}{2} \quad y(\frac{\pi}{4}) = 3 + \sqrt{2}$

$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = \frac{\sqrt{2}}{\frac{3\sqrt{2}}{2}} = \frac{2}{3}$

$y - (3 + \sqrt{2}) = \frac{2}{3}\left(x - \left(2 - \frac{3\sqrt{2}}{2}\right)\right)$

c) y-intercept : $x = 0$

$0 = 2 - 3\cos t$

$3\cos t = 2$

$\cos t = 2/3$

$t = -0.841 \quad t = 0.841$

$L = \int_{-0.841}^{0.841} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= 3.756$

6) $x = \sin t \quad y = \csc t$

$\frac{dx}{dt} = \cos t \quad \frac{dy}{dt} = -\csc t \cot t$

$\frac{dy}{dx} = \frac{-\csc t \cot t}{\cos t} = \frac{-\frac{1}{\sin t} \cdot \frac{\cos t}{\sin t}}{\cos t}$

$= -\frac{1}{\sin^2 t} = -\csc^2 t$

$\frac{d^2y}{dx^2} = \frac{-2\csc t(-\csc t \cot t)}{\cos t}$

$= \frac{2\csc^2 t \cot t}{\cos t} = 2\csc^3 t$

$\frac{d^2y}{dx^2} = 2\csc^3 t$

$-$

$0 \qquad \frac{\pi}{2}$

$+$

$0 \qquad \pi/2$

DEC & concave up